

## 4061- Lecture Six

### Outline

Mirrorless Laser

Optical Feedback

$\gamma_{\text{threshold}}$  for cavity

Population inversion density requirements for lasing

$\gamma \geq \gamma_{\text{threshold}}$

Requirements for Laser

Good Cavity/Fabry-Perot

Optically pumped gas laser

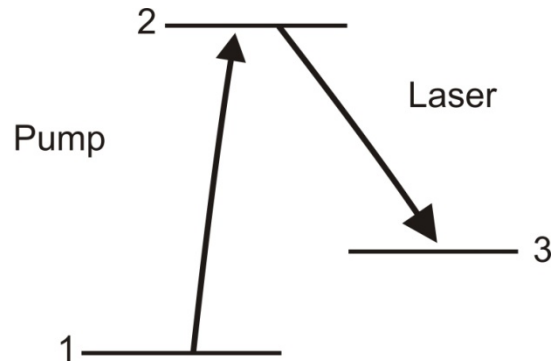
He-Ne Laser

## Recall

$$I = I_0 e^{\gamma z}$$
$$\gamma = \Delta n \sigma = \Delta n A_{21} (\lambda^2 / 8\pi) g(\nu_L)$$

- Independent of intensity
- “small signal gain”
- What is the amplification if  $\gamma > 0$

## Mirrorless Laser



- Pulsed excitation to transfer population from 1 to 2 in cylindrical column
- Photons emitted (due to spontaneous emission) on the  $2 \rightarrow 3$  transition along long axis are amplified
- Direction of output and beam size are related to sample size and shape

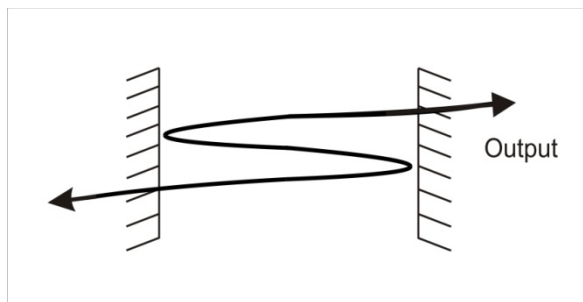
### Example

$$\gamma = 1 \times 10^{-2} / \text{cm} = 1 / \text{m}$$

$$L = 1 \text{ m}$$

$$e^{\gamma L} = e^{(1)(1)} = 2.7$$

- Incident intensity ( $I_0$ ) is amplified by a factor of 2.7



## Feedback

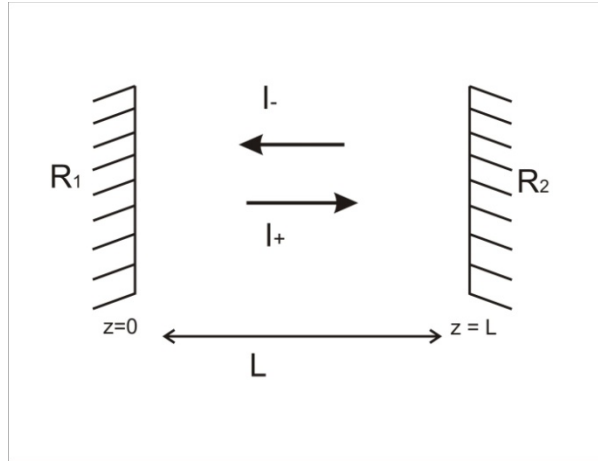
- To increase amplification, use optical feedback with mirrors

## In Practice

- Amplification must overcome losses due to
  - o Diffraction
  - o Mirror absorption/mirror scattering
  - o Coupling losses

So let us assume that medium absorption and medium scattering is small

## Cavity Threshold to Sustain Laser Oscillation



- Incident Light -  $I$
- Reflected Light -  $RI$
- Transmitted Light -  $TI$

$$R = r^2$$

$$T = t^2$$

- $R$  and  $T$  are Reflection and Transmission coefficients for intensity
- $r$  are the reflection and transmission coefficients for E field

$$E_{\text{ref}} = r E_{\text{incident}}$$

$$E_{\text{trans}} = t E_{\text{incident}}$$

## Conservation of Energy

$$R + T = 1$$

$$R + T + S = 1$$

- $S$  is scattering loss/mirror absorption

Near Threshold  $\rightarrow$  Consider Roundtrip of EM wave

$$dI^+/dz = \gamma I^+ \rightarrow I^+(z) = I^+(z=0)e^{\gamma z}$$

$$dI^-/dz = -\gamma I^- \rightarrow I^-(z) = I^-(z=L)e^{\gamma(L-z)}$$

$z = L$

$$\Gamma^+(L) = \Gamma^+(0)e^{\gamma z} \quad (1)$$

$z = 0$

$$\Gamma(0) = \Gamma(L)e^{\gamma z} \quad (2)$$

### Boundary Condition

$$\Gamma^+(0) = \Gamma(0)R_1 \quad (3)$$

Using (2) gives

$$\Gamma^+(0) = R_1[e^{\gamma L} \Gamma(L)] \quad (4)$$

### Boundary Condition

$$\Gamma(L) = \Gamma^+(L)R_2 \quad (5)$$

Using (4) and (5) gives

$$\Gamma^+(0) = R_1 e^{\gamma L} [\Gamma^+(L)R_2] = R_1 R_2 e^{\gamma L} [\Gamma^+(0)e^{\gamma z}] = R_1 R_2 e^{2\gamma L} \Gamma^+(0)$$

Using (1) under the condition that  $\Gamma^+(0)$  is unchanged in 1 Roundtrip

$$R_1 R_2 e^{2\gamma L} = 1$$

This is the Condition for sustaining laser oscillation in cavity

$$\Gamma^+(0) \neq 0$$

Include Medium/Mirror Losses

$$R_1 R_2 e^{2(\gamma-\alpha)L} \geq 1$$
$$2(\gamma-\alpha)L = \ln(1/R_1 R_2)$$

- Cavity threshold gain  $\gamma_{th} = \alpha + (1/2L)(\ln(1/R_1 R_2))$
- Large R  $\rightarrow$  small  $\gamma_{th}$
- In practice we need  $\gamma \gg \gamma_{th}$  ie. Atomic gain should overcome cavity gain

$$\Delta n A_{21} (\lambda^2 / 8\pi) g(\nu) \gg \gamma_{th}$$
$$\Delta n \geq 8\pi \gamma_{th} / A_{21} \lambda^2 g(\nu)$$

This is the inversion density required for lasing

$\Delta n$  should be small since large  $\Delta n$  means more pump power

Substitute value for  $g(\nu_0) = g_{max} = 2/\pi\Delta\nu$

$$\Delta n \geq 8\pi \gamma_{th} \pi\Delta\nu / 2A_{21}\lambda^2$$

Requirements for Reducing  $\Delta n$

Need low  $\alpha_{cavity}$

High  $R_1 R_2 \rightarrow$  small  $\gamma_{th}$

Long cavity  $\rightarrow$  small  $\gamma_{th}$

Narrow gain linewidth

Large  $A_{21}$