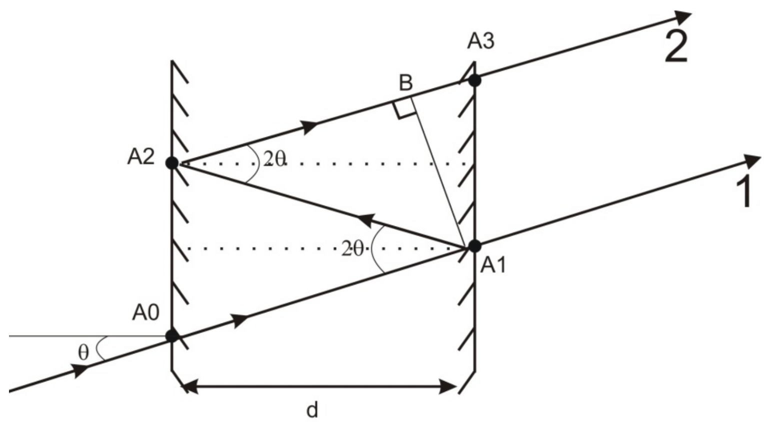
4061- Lecture Seven

Fabry-Perot Interferometers (FPI) can be used as high resolution instruments in spectroscopy. The principles associated with a FPI are also the basis for understanding a laser cavity (resonator).

Properties

Ensures that transmission function has narrow frequency range (for high reflectivity mirrors)
Transmission peaks correspond to EM modes sustained by cavity



Interference Condition

Highly reflective mirrors are coated on inner surface.

θ = angle of incidence (usually close to normal incidence)
d = mirror spacing that defines dimensions of 1D cavity

Note: Beams 1 and 2 can be focused using a lens on an optical detector used to measure light transmitted by the cavity.

p = path difference between beams 1 and 2 = $A_1A_2 + A_2B$

$$A_2B = A_1A_2 \cos 2\theta$$

$$p = A_1A_2[1 + \cos 2\theta] = A_1A_2[2 \cos^2 \theta]$$

since $d = A_1A_2\cos\theta$, $p = 2d\cos\theta$

Note: when θ = 0 p = 2d (normal incidence)

Constructive Interference means $p = m\lambda$ or $2d\cos\theta = m\lambda$
For θ = 0̊, $d = \frac{m\lambda}{2}$ which defines condition for maximum transition

Define

- I_o = incident intensity which is proportional to square of incident E field amplitude
- t, t'= transmission coefficients for amplitude at left and right mirrors
- r = reflection coefficient for amplitude at either mirror surface

- $R = r^2$ = intensity reflection coefficient

Assume incident electric field is $E_o e^{iwt}$

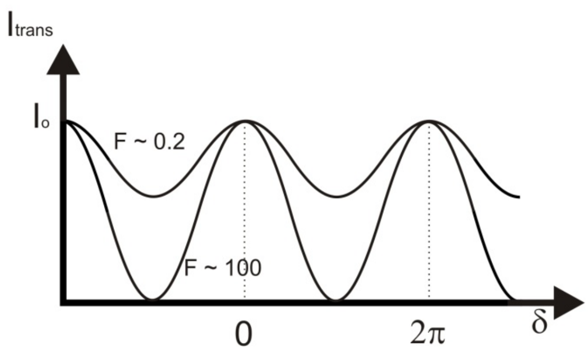
N=1 $E_1 = E_o tt' e^{i\omega t}$ 1st transmitted beam
N=2 $E_2 = E_o trrt' e^{i(\omega t-\delta)}$ 1st transmitted beam
N=3 $E_3 = E_o tr^4 t' e^{i(\omega t-\delta)}$ 1st transmitted beam
N $E_N = E_o tr^{2(N-1)} t' e^{i(\omega t-(N-1)\delta)}$ 1st transmitted beam

- Phase shift → $e^{-i\delta}$ per round trip
  $\delta = \frac{2\pi}{\lambda} p$
- Note: two reflections per round trip means that phase change is about 2π
- Effect not important since we want intensity as a function of d

$$E_{total} = E_o tt' e^{i\omega t}/(1-r^2 e^{-i\delta})$$
$$I_{trans} \propto E_T E_T^* = I_o (tt')^2/(1+r^4 - 2r^2 \cos\delta)$$

Define Coefficient of Finesse

$$F = 4r^2/(1-r^2)^2 = 4R/(1-R)^2$$
$$I_{trans} = I_o/(1+(2r/(1-r^2)^2 \sin^2\delta/2 = I_o/(1+F\sin^2\delta/2)$$



δ is proportional to p since $\delta = \frac{2\pi}{\lambda} p$

Maxima
$\delta = 0, 2\pi, \dots, 2m\pi$ => $\sin(\delta/2) = 0$

Minima
$\delta = \pi, 3\pi \dots$ => $\sin(\delta/2) = 1$

- if r ~ 1, F is large (10^4- 10^5) is that I_{trans} at minima is ~ 0

Estimate FWHM of Fringe
$$0.5 I_o = I_o / (1+F\sin^2(\delta/2))$$
$$\sin(\delta/2) = 1/\sqrt{F}$$
$$\delta = 2/\sqrt{F} \text{ (using small angle approximation)}$$

$$\delta_{FWHM} = 4/\sqrt{F}$$

FWHM is related to resolution

- δ_{min} is the minimum phase change that can be measured

Define Finesse $\mathcal{F}$

$$\frac{\text{Separation of adjacent maxima}}{\text{FWHM}} = \frac{2\pi}{\frac{4}{\sqrt{F}}} = \frac{\pi\sqrt{F}}{2} = \mathcal{F}$$

$\mathcal{F}$ is related to the number of round trips
$\mathcal{F} \sim 30$ for good (He-Ne laser) cavity
$\mathcal{F} \sim 10^4$ for confocal FPI

Laser Resonator